Pearson
Edexcel

Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 02 Core Pure Mathematics 2

Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a) (i) (a) (ii)	$\{arg(z_1) =\}\, tan^{-1}\left(\frac{-3}{3}\right)$ or $\{arg(z_1) =\}\, tan^{-1}(-1)$ or $\{arg(z_1) =\} - tan^{-1}\left(\frac{3}{3}\right)$ or $\{arg(z_1) =\} -\frac{\pi}{4}$ or $\{arg(z_1) =\} 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ or states should be -3 not 3 on top	B1	2.3
	States that $\left\{arg\left(\frac{z_1}{z_2}\right) =\right\} arg(z_1) - arg(z_2)$ Or states that the arguments should be subtracted	B1	2.3
		(2)	
(b)	$\left\{arg\left(\frac{z_1}{z_2}\right) = \left(\text{their} -\frac{\pi}{4}\right) - \frac{\pi}{6} =\right\} -\frac{5\pi}{12}$ Or $\left\{arg\left(\frac{z_1}{z_2}\right) = \left(\text{their}\, \frac{7\pi}{4}\right) - \frac{\pi}{6}\right\} = \frac{19\pi}{12}$	B1ft	2.2a
		(1)	
		(3 marks)	

Notes:

(a) (i)

B1: See scheme, Condone – 45

Any incorrect arguments seen is B0.

$arg(z_1) = tan^{-1}\left(\frac{3}{-3}\right)$ is B0

Note: They used 3 instead of -3 is B0, there are two 3's in line 1 do they mean both should -3

It should be negative is B0

(a) (ii)

B1: See scheme

(b)

B1ft: States a correct value for $arg\left(\frac{z_1}{z_2}\right)$ Follow through on their answer to part (a) (i), do not ISW

Question	Scheme	Marks	AOs
2(a)(i)	x / C = number of **Construction** students y / D = number of **Design** students z / H = number of **Hospitality** students	B1	3.3
(ii)	The increase in number of students in 2020 $1110 \times 0.0027\{= 2.997 \approx 3\}$ Or The number of students in 2020 $1110 \times 1.0027 = \{1112.997 \approx 1113\}$	M1	1.1b
	$x + y + z = 1110$ $\quad C + D + H = 1110$ $x - z = 370$ o.e. $\quad C - H = 370$ o.e. $0.0125C + 0.025D - 0.02H = 3$or 2.997 o.e $1.0125C + 1.025D + 0.98H = 1113$ or 1112.997o.e. $0.0125x + 0.025y - 0.02z = 3$ or 2.997o.e$1.0125x + 1.025y + 0.98z = 1113$ or 1112.997 o.e.	M1 A1	3.3 1.1b
		(4)	
(b)	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1.0125 & 1.025 & 0.98 \end{pmatrix}\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1110 \\ 370 \\ 1113 \end{pmatrix}$ or $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0.0125 & 0.025 & -0.02 \end{pmatrix}\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1110 \\ 370 \\ 3 \end{pmatrix}$	M1 A1ft	1.1b 1.1b
	$\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1.0125 & 1.025 & 0.98 \end{pmatrix}^{-1}\begin{pmatrix} 1110 \\ 370 \\ 1113 \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}$ or $\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0.0125 & 0.025 & -0.02 \end{pmatrix}^{-1}\begin{pmatrix} 1110 \\ 370 \\ 3 \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}$	dM1	1.1b
	So in 2019, **720** students studied **C**onstruction, **40** students studied **D**esign and **350** students studied **H**ospitality	A1	3.2a
		(4)	
			(8 marks)

Notes:

Mark (i) and (ii) together

(a)(i)

B1: Defines 3 variables, minimum e.g. construction = C, Design = D, Hospitality = H. This may be seen in text of the question, abbreviations may be used

(ii)

M1: Finds either the increase or the number of students in 2020. This may be implied by any equation which equals 1113 or 1112.997. If students use 1100 instead of 1110 this is slip and we can award this mark.

M1: Attempts to use the model to set up at least 2 equations

A1: All 3 simplified equations correct (decimals or fractions), one for each different piece of information. Award with mark even if B0 is scored and it is clear what the variables used stand for. Ignore any additional equations even if incorrect. As soon as 3 correct equations are seen you may award this mark.

<u>Alternative approach</u>

(i) B1: Construction = $H + 370$, Design = D, Hospitality = H

(ii) M1M1A1: $H + 370 + D + H = 1110$ o.e $C = H + 3701.0125(H + 370) + 1.025D + 0.98H = 1113$ or 1112.997 o.e. they do not need to be simplified

(b) This is M1 M1 A1 A1 on ePen but is marked M1A1M1A1

M1: Uses their equation in part(a) to set up a matrix equation of the form $\begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix}\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \cdots \end{pmatrix}$, where "…" are numerical values.

A1ft: Correct matrix equation for their equations

dM1: Dependent on previous method mark. Writes $\left(\text{their } A()^{-1}\begin{pmatrix} 1110 \\ \text{their "370"} \\ \text{their "3"} \end{pmatrix}\right)$ and obtains at least one value of C, D or H. The inverse matrix need not be found, writing $\mathbf{A}^{-1}\begin{pmatrix} 1110 \\ 370 \\ \text{their "3"} \end{pmatrix} = \ldots$ is sufficient. A correct matrix equation followed by correct values implies this mark.

Condone $\begin{pmatrix} 1110 \\ \text{their "370"} \\ \text{their "3"} \end{pmatrix}\mathbf{A}^{-1} = \ldots$ as long as they reach some values. The values imply the correct method

Note: $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0.0125 & 0.025 & -0.02 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{10}{23} = 0.43\ldots & \frac{18}{23} = 0.78\ldots & -\frac{400}{23} = -17.39\ldots \\ \frac{3}{23} = 0.13\ldots & -\frac{13}{23} = -0.56\ldots & \frac{800}{23} = 34.78\ldots \\ \frac{10}{23} = 0.43\ldots & -\frac{5}{23} = -0.21\ldots & -\frac{400}{23} = -17.39\ldots \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1.0125 & 1.025 & 0.98 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{410}{23} = 17.82\ldots & \frac{18}{23} = 0.78\ldots & -\frac{400}{23} = -17.39\ldots \\ -\frac{797}{23} = -34.65\ldots & -\frac{13}{23} = -0.56\ldots & \frac{800}{23} = 34.78\ldots \\ \frac{410}{23} = 17.82\ldots & -\frac{5}{23} = -0.21\ldots & -\frac{400}{23} = -17.39\ldots \end{pmatrix}$$

A1: Interprets the answer in the context of the question, minimum is $C = 720$, $D = 40$, $H = 350$ with their variables. Condone the variable not been defined for this mark if it is clear which variable belong to what course.

Note: they must be using a matrix equation to solve the equation to score any marks.

Alternative approach

For example

Equations simplifies to $C - H = 370$, $D + 2H = 740$ and $1.025D + 1.9925H = 738.375$

which leads to $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 0 & 1.025 & 1.9925 \end{pmatrix}\begin{pmatrix} C \\ D \\ H \end{pmatrix} = \begin{pmatrix} 740 \\ 370 \\ 738.375 \end{pmatrix}$ then $\begin{pmatrix} C \\ D \\ H \end{pmatrix} =$

$$\begin{pmatrix} 17.826 & 1 & -17.3913 \\ -34.6521 & 0 & 34.7826 \\ 17.826 & 0 & -17.3913 \end{pmatrix}\begin{pmatrix} 740 \\ 370 \\ 738.375 \end{pmatrix} = \begin{pmatrix} 720 \\ 40 \\ 350 \end{pmatrix}$$

Note: A 2 x 2 matrix is fine if it is appropriate for their equation.

Special Case: Forming an equation in one variable

(a)(i) B1: Hospitality = x, Construction = x + 370, Design = 740 – 2x

(ii) M1M1A1: $1.0125(x + 370) + 1.025(740 - 2x) + 0.98x = 1113$ or 1112.997

(a)(i) B1: Hospitality = x – 370, Construction = x , Design = 1480 – 2x

(ii) M1M1A1: $1.0125(x) + 1.025(1480 - 2x) + 0.98(x - 370) = 1113$ or 1112.997

(b) M0A0M0A0: They have an equation and are not forming and solving a matrix equation

Question	Scheme	Marks	AOs
3(a)	$n=1 \Rightarrow \ \mathbf{M}^1 = \begin{pmatrix} 3^1 & \frac{a}{2}(3^1-1) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$ {So the result is true for $n=1$}	B1	2.2a
	Assume true for $n=k$ Or assume $\mathbf{M}^n$ or $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & \frac{a}{2}(3^k-1) \\ 0 & 1 \end{pmatrix}$	M1	2.4
	A correct method to find an expression for $n=k+1$ $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3^k & \frac{a}{2}(3^k-1) \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3^k & \frac{a}{2}(3^k-1) \\ 0 & 1 \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} 3(3^k) & a(3^k)+\frac{a}{2}(3^k-1) \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3(3^k) & 3\times\frac{a}{2}(3^k-1)+a \\ 0 & 1 \end{pmatrix}$	A1	1.1b
	$\begin{pmatrix} 3^{k+1} & \frac{a}{2}[2(3^k)+(3^k-1)] \\ 0 & 1 \end{pmatrix} =$ $\begin{pmatrix} 3^{k+1} & \frac{a}{2}[3(3^k)-1] \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3^{k+1} & \frac{a}{2}[3^{k+1}-1] \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 3(3^k) & 3\times\frac{a}{2}(3^k-1)+a \\ 0 & 1 \end{pmatrix} =$ $\begin{pmatrix} 3^{k+1} & \frac{a}{2}(3(3^k-1)+2) \\ 0 & 1 \end{pmatrix}$ $= \begin{pmatrix} 3^{k+1} & \frac{a}{2}(3^{k+1}-1) \\ 0 & 1 \end{pmatrix}$	A1	2.1
	If true for $n=k$ **then** true for $n=k+1$ and as it is **true** for $n=1$ the statement is **true for** all (positive integers) $\boldsymbol{n}$	A1	2.4
		(6)	
(b)(i)	$det(\mathbf{M}^n) = 3^n$ or $det(\mathbf{M}) = 3$	B1	1.1b
	Uses $5\times det(\mathbf{M}^n) = 1215 \Rightarrow p^n = q \Rightarrow n = \ldots$ $5\times 3^n = 1215 \Rightarrow 3^n = 243 \Rightarrow n = \ldots$	M1	3.1a
	$n=5$	A1	1.1b
(ii)	$\begin{pmatrix} 3^n & \frac{a}{2}(3^n-1) \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 123 \\ -2 \end{pmatrix} \Rightarrow 2(3^n) - 2\frac{a}{2}(3^n-1) = 123$ $\Rightarrow a = \ldots$	M1	1.1b

	$\begin{pmatrix} 243 & \frac{a}{2}(243-1) \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 123 \\ -2 \end{pmatrix} \Rightarrow 2(243) - 2\frac{a}{2}(243-1) = 123 \Rightarrow a = \ldots$ $\frac{1}{243}\begin{pmatrix} 1 & -\frac{a}{2}(243-1) \\ 0 & a \end{pmatrix}\begin{pmatrix} 123 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} \Rightarrow \frac{123 - 2\frac{a}{2}(243-1)}{243} = -2 \Rightarrow a = \ldots$		
	$a = 1.5$	A1	1.1b
		(5)	
(11 marks)			

Notes:

(a)

B1: Shows that the result holds for $n = 1$. Must see substitution in the RHS minimum required is $\begin{pmatrix} 3 & \frac{a}{2}(3-1) \\ 0 & 1 \end{pmatrix}$ and reaches $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$

M1: Assumes the result is true for some value of $n = k$. Assume (true for) $n = k$ is sufficient.

Alternatively states assume $\mathbf{M}^n$ or $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & \frac{a}{2}(3^k-1) \\ 0 & 1 \end{pmatrix}$

M1: Sets up a matrix multiplication of their assumed result multiplied by the original matrix, either way round. Allow a slip as long as the intention is clear.

A1: Achieves a correct un-simplified matrix

A1: Reaches a correct simplified matrix with **no errors, the correct un-simplified matrix seen previously and at least one intermediate line which must be correct.**

A1: Correct conclusion. This mark is dependent on all previous marks except B mark but $n = 1$ must have been attempted. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. Condone $n \in \mathbb{Z}$

(b)(i)

B1: States correct determinant. This can be implied by a correct equation

M1: Correct method to find a value of n using $5 \times$ 'their $det(\mathbf{M}^n)$' $= 1215$ which involves solving an index equation of the form $p^n = q$ where $n > 1$

A1: $n = 5$

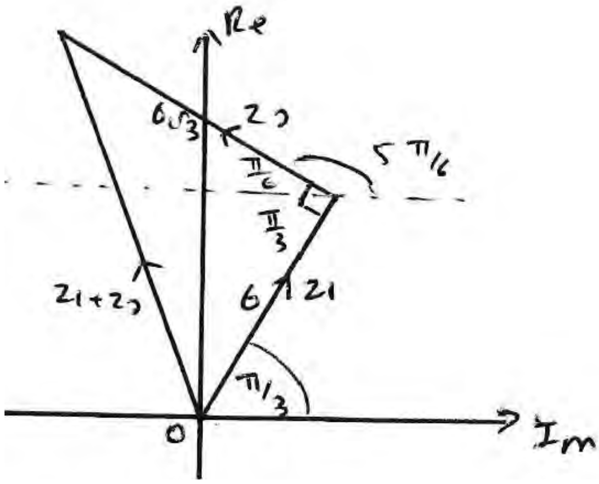
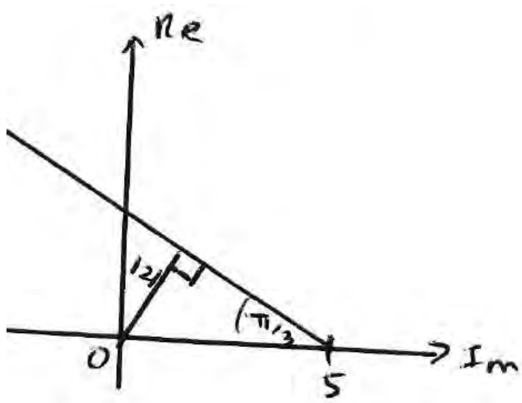
(ii)

M1: Sets up an equation by multiplying the matrix $\mathbf{M}^n$ by $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ setting equal to $\begin{pmatrix} 123 \\ -2 \end{pmatrix}$ and reaches a value for a. You may just see $2(3^n) - 2\frac{a}{2}(3^n - 1) = 123 \Rightarrow a = \ldots$

Follow through on their value for n.

A1: $a = 1.5$

Question	Scheme	Marks	AOs
4(i)	$z_1 = 6\left[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right] = \ldots\{3 + 3\sqrt{3}i\}$ $z_2 = 6\sqrt{3}\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right] = \ldots\{-9 + 3\sqrt{3}i\}$ $\{z_1 + z_2 =\}(3 + 3\sqrt{3}i) + (-9 + 3\sqrt{3}i) = \ldots\{-6 + 6\sqrt{3}i\}$ Or $\{z_1 + z_2 =\}6\left[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right] + 6\sqrt{3}\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right] = a + bi$ where a and b are constants, the trig function must be evaluated	M1	3.1a
	Clearly show the method to find modulus **and** argument for $z_1 + z_2$ $arg(z_1 + z_2) = \pi - \tan^{-1}\left(\frac{6\sqrt{3}}{6}\right)$ or $\tan^{-1}\left(\frac{6\sqrt{3}}{-6}\right) = \ldots\left\{\frac{2\pi}{3}\right\}$ **and** $\lvert z_1 + z_2\rvert = \sqrt{6^2 + (6\sqrt{3})^2} = \ldots\{12\}$ **Alternative 1** $-6 + 6\sqrt{3}i = 12\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ $= 12\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$ **Alternative 2** $12e^{\frac{2\pi}{3}i} = 12\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = \ldots\{-6 + 6\sqrt{3}i\}$	dM1	2.1
	$z_1 + z_2 = 12e^{\frac{2\pi}{3}i}$ * $12e^{\frac{2\pi}{3}i} = -6 + 6\sqrt{3}i$ Therefore $z_1 + z_2 = 12e^{\frac{2\pi}{3}i}$*	A1*	1.1b
		(3)	
	Alternative 3 $z_1 + z_2 = 6e^{\frac{\pi}{3}i} + 6\sqrt{3}e^{\frac{5\pi}{6}i}$ $= 12\left[\frac{1}{2}\cos\left(\frac{\pi}{3}\right) + \frac{1}{2}i\sin\left(\frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}\cos\left(\frac{5\pi}{6}\right) + \frac{\sqrt{3}}{2}i\sin\left(\frac{5\pi}{6}\right)\right]$	M1	3.1a
	$12\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 12\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$	dM1	2.1
	$z_1 + z_2 = 12e^{\frac{2\pi}{3}i}$*	A1*	1.1b
		(3)	
	Alternative 4 $z_1 + z_2 = 6e^{\frac{\pi}{3}i} + 6\sqrt{3}e^{\frac{5\pi}{6}i} = 6e^{\frac{\pi}{3}i}\left(1 + \sqrt{3}e^{\frac{\pi}{2}i}\right) = 6e^{\frac{\pi}{3}i}(1 + \sqrt{3}i)$	M1	
	Either $r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$ **and** $arg = \arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$	dM1	

	Or $6e^{\frac{\pi}{3}i}(1+\sqrt{3}i) = 12e^{\frac{\pi}{3}i}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i()e^{\frac{\pi}{3}i}\left(cos\left(\frac{\pi}{3}\right)+i\,sin\left(\frac{\pi}{3}\right)\right)\right)$		
	$z_1 + z_2 = 12e^{\frac{\pi}{3}i}e^{\frac{\pi}{3}i} = 12e^{\frac{2\pi}{3}i}*$	A1*	
		(3)	
	Alternative 5 Uses geometry to show that $z_1,\ z_2$ and z_1+z_2 form a right-angled triangle 	M1	3.1a
	$arg(z_1+z_2) = \frac{\pi}{3} + tan^{-1}\left(\frac{6\sqrt{3}}{6}\right) = \ldots\left\{\frac{2\pi}{3}\right\}$ $\lvert z_1+z_2\rvert = \sqrt{(6)^2+(6\sqrt{3})^2} = \ldots\{12\}$	dM1	1.1b
	$z_1 + z_2 = 12e^{\frac{2\pi}{3}i}$ *	A1*	1.1b
		(3)	
(ii)		M1	3.1a
	$sin\left(\frac{\pi}{3}\right) = \frac{\lvert z\rvert}{5} \Rightarrow \lvert z\rvert = \ldots$	M1	1.1b
	$\lvert z\rvert = \frac{5\sqrt{3}}{2}$	A1	1.1b
		(3)	

	Alternative 1 Gradient = $-\tan\left(\frac{\pi}{3}\right)$ $c = 5\tan\left(\frac{\pi}{3}\right)$ leading to $y = -\sqrt{3}x + 5\sqrt{3}$ or $\tan\left(\frac{\pi}{3}\right) = \frac{y}{5-x}$ $\lvert z\rvert^2 = x^2 + y^2 = x^2 + (-\sqrt{3}x + 5\sqrt{3})^2 = 4x^2 - 30x + 75$ $\frac{d\lvert z\rvert^2}{dx} = 8x - 30 = 0 \Rightarrow x = \ldots\{3.75\}$ or $\lvert z\rvert^2 = 4(x - 3.75)^2 + 18.75 \Rightarrow x = \ldots\ldots\{3.75\}$	M1	3.1a
	$\lvert z\rvert = \sqrt{4(\text{their}3.75)^2 - 30(\text{their}3.75) + 75}$	M1	1.1b
	$\lvert z\rvert = \frac{5\sqrt{3}}{2}$	A1	1.1b
		(3)	
	Alternative 2 Gradient = $-\tan\left(\frac{\pi}{3}\right)$ $c = 5\tan\left(\frac{\pi}{3}\right)$ leading to $y = -\sqrt{3}x + 5\sqrt{3}$ Perpendicular line through the origin $y = \frac{1}{\sqrt{3}}x$ and find the point of intersection of the two lines $\left(\frac{15}{4}, \frac{5\sqrt{3}}{4}\right)$	M1	3.1a
	Finds the distance from the origin to their point of intersection $\lvert z\rvert = \sqrt{\left(\text{their } \frac{15}{4}\right)^2 + \left(\text{their } \frac{5\sqrt{3}}{4}\right)^2} = \ldots$	M1	1.1b
	$\lvert z\rvert = \frac{5\sqrt{3}}{2}$	A1	1.1b
		(3)	
(6 marks)			

Notes:

(i)

M1: A complete method to find both $z_1$and z_2 in the form $a + bi$and adds them together.

dM1: Dependent on previous method mark, finds the modulus and argument of $z_1 + z_2$. They must show their method, just stating modulus = 12 and argument = $\frac{2\pi}{3}$ is not sufficient as this is a show question.

Alternative 1: Factorises out 12 and find the argument

Alternative 2: uses $12e^{\frac{2\pi}{3}i} = 12\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = \ldots$

A1*: Achieves the correct answer following no errors or omissions**.**

Alternatively shows that $12e^{\frac{2\pi}{3}i} = -6 + 6\sqrt{3}i$and concludes therefore $z_1 + z_2 = 12e^{\frac{2\pi}{3}i}*$

<u>**Alternative 3**</u>

M1: Factorises out 12 and writes in the form $12\left[\ldots \boldsymbol{cos}\left(\frac{\pi}{3}\right)+\ldots \boldsymbol{i\,sin}\left(\frac{\pi}{3}\right)+\ldots \boldsymbol{cos}\left(\frac{5\pi}{6}\right)+\ldots \boldsymbol{i\,sin}\left(\frac{5\pi}{6}\right)\right]$ **dM1:** Dependent on previous mark**.** Writes in the form $12(a+bi)$ leading to the form $12(cos\,\theta + i\,sin\,\theta)$ **A1*:** Achieves the correct answer following no errors or omissions**.** **<u>Alternative 4</u>** **M1:** Factorises out 6 and writes in the form $6e^{\frac{\pi}{3}i}\left(1+\sqrt{3}e^{\frac{\pi}{2}i}\right)=6e^{\frac{\pi}{3}i}(1+ai)$ **dM1:** Dependent on previous method mark, finds the modulus and argument of $(1+ai)$ or $12(a+bi)$ leading to the form $12(cos\,\theta + i\,sin\,\theta)$ **A1*:** Achieves the correct answer following no errors or omissions**.** **<u>Alternative 5</u>** **M1:** Draws a diagram to show that z_1, z_2 and z_1+z_2 form a right-angled triangle. **dM1:** Dependent on previous method mark, finds the modulus and argument of z_1+z_2 **A1*:** Achieves the correct answer following no errors or omissions**.** **Note:** Writing $arg(z_1+z_2)=arctan\left(\frac{6\sqrt{3}}{-6}\right)=-\frac{\pi}{3}$ therefore $arg(z_1+z_2)=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$ with no diagram or finding z_1+z_2 is **M0dM0A0**
(ii) **M1**: Draws a diagram and recognises that the shortest distance will form a right-angled triangle. **M1:** Uses trigonometry to find the shortest length. **A1:** Correct exact value. **<u>Alternative 1</u>** **M1**: Finds the equation of the half-line by attempting $m=-\,tan\left(\frac{\pi}{3}\right)$ $c=5\,tan\left(\frac{\pi}{3}\right)$. Finds x^2+y^2 in terms of *x*, differentiates, sets = 0 and finds the value of *x*. **M1:** Uses their value of *x* to find the minimum value of $\sqrt{x^2+y^2}$ **A1:** Correct exact value. **<u>Alternative 2</u>** **M1**: Finds the equation of the half-line by attempting $m=-\,tan\left(\frac{\pi}{3}\right)$ $c=5\,tan\left(\frac{\pi}{3}\right)$. Finds the equation of the line perpendicular which passes through the origin. Finds the point of intersection of the lines **M1:** Finds the distance from the origin to their point of intersection **A1:** Correct exact value.

Question	Scheme	Marks	AOs
5(a)	$\sin y = x \Rightarrow \cos y \dfrac{dy}{dx} = 1$ $\sin y = x \Rightarrow \dfrac{dx}{dy} = \cos y$	M1	1.1b
	Uses $\sin^2 y + \cos^2 y = 1 \Rightarrow \cos y = \sqrt{1-\sin^2 y} \Rightarrow \sqrt{1-x^2}$	M1	2.1
	$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ * cso	A1*	1.1b
		(3)	
(b)	Using the answer to (a) $f'(x) = \dfrac{1}{\sqrt{1-e^{2x}}} \times \ldots$ Restart $\sin y = e^x \Rightarrow \cos y \dfrac{dy}{dx} = e^x$	M1	3.1a
	$f'(x) = \dfrac{1}{\sqrt{1-e^{2x}}} \times e^x$ $f'(x) = \dfrac{e^x}{\cos y}$	A1	1.1b
	$e^x \neq 0$ (or $e^x > 0$) therefore, there are no stationary points Alternatively, $e^x = 0$ leading to $x = \ln 0$ which is impossible/undefined therefore there are no stationary points.	A1	2.4
		(3)	
		(6 marks)	

Notes:

(a)

M1: Finds x in terms of y and differentiates

M1: Uses the trig identity $\sin^2 y + \cos^2 y = 1$ to express $\cos y$ in terms of x. This may be seen in their derivative or stated on the side

A1*: Correctly achieves the printed answer $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$. cso

(b)

M1: Differentiates using the chain rule to achieve the correct form, condone $f'(x) = \frac{1}{\sqrt{1-e^{2x}}}$

Note $f'(x) = \frac{1}{\sqrt{1-e^{x}}}$ is B0 for incorrect form

Alternatively restart, finds x in terms of y and differentiates

A1: Correct differentiation

A1: Follows correct differentiation. States that as $e^x \neq 0$ (or $e^x > 0$) or no solutions to $e^x = 0$ therefore there are no stationary points.

Alternatively, $e^x = 0$ leading to $x = \ln 0$ which is impossible/undefined/error therefore there are no stationary points. Ignore any reference to the denominator = 0

Question	Scheme	Marks	AOs
6(a)	$4x^3+px^2-14x+q=0 \Rightarrow x^3+\frac{p}{4}x^2-\frac{14}{4}x+\frac{q}{4}=0$ $\alpha+\beta+\gamma=-\frac{p}{4}\ \ \alpha\beta+\alpha\gamma+\beta\gamma=-\frac{14}{4}$ or $-\frac{7}{2}$	B1	3.1a
	$(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\alpha\gamma+\beta\gamma)$ $\left(-\frac{p}{4}\right)^2=16+2\left(-\frac{7}{2}\right)\Rightarrow p=\ldots$ or $(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)=\alpha^2+\beta^2+\gamma^2$ $\left(-\frac{p}{4}\right)^2-2\left(-\frac{7}{2}\right)=16\Rightarrow p=\ldots$	M1	3.1a
	$p=12$ * cso	A1*	1.1b
		(3)	
(b)	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$	M1	1.1b
	$\frac{\left(-\frac{7}{2}\right)}{\left(\frac{-q}{4}\right)}=\frac{14}{3}\Rightarrow q=\ldots$	M1	1.1b
	$q=3$	A1	1.1b
		(3)	
	Alternative $4(\frac{1}{w})^3+12(\frac{1}{w})^2-14(\frac{1}{w})+q\{=0\}$	M1	1.1b
	$qw^3-14w^2+12w+4=0\Rightarrow\frac{14}{3}=-\frac{-14}{q}\Rightarrow q=\ldots$	M1	1.1b
	$q=3$	A1	1.1b
		(3)	
(c)	$(\alpha-1)(\beta-1)(\gamma-1)=\ldots$ $=\alpha\beta\gamma-(\alpha\beta+\alpha\gamma+\beta\gamma)+(\alpha+\beta+\gamma)-1$	M1 A1	1.1a 1.1b
	$=\left(-\frac{\text{their }3}{4}\right)-\left(-\frac{7}{2}\right)+\left(-\frac{12}{4}\right)-1=\ldots$	dM1	1.1b
	$=-\frac{5}{4}$	A1	1.1b
		(4)	
Alt	$4(x+1)^3+12(x+1)^2-14(x+1)+'3'\{=0\}$ or substitutes in 1	M1	1.1a
	$=\ldots4+\ldots12+\ldots-14+'3'=5$ or $4x^3+24x^2+22x+2+$ 'their q'	A1ft	1.1b

	$= -\frac{\text{'their constant'}}{4}$	dM1	1.1b
	$= -\frac{5}{4}$	A1	1.1b
(10 marks)			

Notes:

(a)
B1: Identifies the correct values for the sum and pair sum. This may be implied by substituting into an equation, it must be clear
M1: Uses the correct identity and values of their sum **and** their pair sum to find a value of p
A1*: $p = 12$ cso there is no need to see a reason

(b)
M1: Establishes a correct identity
M1: Uses their identity and their pair sum and their product of roots to find a value of q. Condone a slip but the intention must be clear.
A1: $q = 3$ Allow this mark from incorrect sign of both pair sum and product

Alternative
M1: Uses $x = \frac{1}{w}$ the substitution
M1: Simplifies to an quartic equation of the form $aw^3 + bw^2 + cw + d = 0$ and uses $\frac{14}{3} = -\frac{b}{a}$ to find a value for q
A1: $q = 3$

(c)
M1: Attempts to multiply out the three brackets.
A1: Correct expansion.
dM1: Dependent on previous method**.** Substitutes in the value of their sum, pair sum and the value of their product as appropriate. Condone a slip but the intention must be clear
A1: Correct value
<u>Alternative</u>
M1: Substitutes $(x + 1)$ or $x = 1$ into the cubic with their value of q. Allow the use of different letters e.g. $(w + 1)$
A1ft: Correct constant terms, follow through on their value of q
dM1: Applies $-\frac{\text{'their constant'}}{4}$
A1: Correct value

Question	Scheme	Marks	AOs
7(a)	$x = r\cos\theta = (1+\tan\theta)\cos\theta = \cos\theta + \sin\theta$ $= \cos\theta + \tan\theta\cos\theta$ $\frac{dx}{d\theta} = \alpha(1+\tan\theta)\sin\theta + \beta\sec^2\theta\cos\theta$ or $\frac{dx}{d\theta} = \alpha\sin\theta + \beta\cos\theta$ $\frac{dx}{d\theta} = \alpha\sin\theta + \beta\sec^2\theta\cos\theta + \delta\tan\theta\sin\theta$	M1	3.1a
	$\frac{dx}{d\theta} = -(1+\tan\theta)\sin\theta + \sec^2\theta\cos\theta$ or $\frac{dx}{d\theta} = -\sin\theta + \cos\theta$ $\frac{dx}{d\theta} = -\sin\theta + \sec^2\theta\cos\theta - \tan\theta\sin\theta$ or $\frac{dx}{d\theta} = -\sin\theta + \sec\theta - \tan\theta\sin\theta$	A1	1.1b
	For example $\left\{\frac{dx}{d\theta} =\right\} -\sin\theta + \cos\theta = 0 \Rightarrow \tan\theta = 1 \Rightarrow \theta = \ldots$ $\left\{\frac{dx}{d\theta} =\right\} -\sin\theta + \cos\theta = 0 \Rightarrow \sin\theta = \cos\theta \Rightarrow \theta = \ldots$ $\left\{\frac{dx}{d\theta} =\right\} -\sin\theta + \cos\theta = \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right) = \theta = \ldots$ or $\left\{\frac{dx}{d\theta} =\right\} -(1+\tan\theta)\sin\theta + \sec^2\theta\cos\theta = 0$ $\Rightarrow -\sin\theta - \frac{\sin^2\theta}{\cos\theta} + \frac{1}{\cos\theta} = 0 \Rightarrow -\sin\theta + \frac{1-\sin^2\theta}{\cos\theta} = 0$ $\Rightarrow -\sin\theta + \cos\theta = 0 \Rightarrow \tan\theta = 1 \Rightarrow \theta = \ldots$ or $\left\{\frac{dx}{d\theta} =\right\} -\sin\theta - \tan\theta\sin\theta + \sec\theta = 0$ $\Rightarrow -\frac{1}{2}\sin 2\theta - \sin^2\theta + 1 = 0 \Rightarrow \sin 2\theta + 2\sin^2\theta - 1 = 1$ $\Rightarrow \sin 2\theta - \cos 2\theta = 1 \Rightarrow \sqrt{2}\sin\left(2\theta - \frac{\pi}{4}\right) = 1 \Rightarrow \theta = \ldots$ or $\left\{\frac{dx}{d\theta} =\right\} -\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) = 0$ $\left\{\frac{dx}{d\theta} =\right\} -\left(1+\tan\left(\frac{\pi}{4}\right)\right)\sin\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = 0$ $\left\{\frac{dx}{d\theta} =\right\} -\sin\left(\frac{\pi}{4}\right) + \sec^2\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) = 0$	dM1	3.1a
	$r = 1 + \tan\left(\frac{\pi}{4}\right) = 2$ therefore $A\left(2, \frac{\pi}{4}\right)$*	A1*	2.1
		(4)	
	Area bounded by the curve $= \frac{1}{2}\int(1+\tan\theta)^2 \ \{d\theta\}$	M1	3.1a

(b)	$=\frac{1}{2}\int(1+2\tan\theta+\tan^2\theta)\ \{d\theta\}$ $=\frac{1}{2}\int(1+2\tan\theta+[\sec^2\theta-1])\ \{d\theta\}=\ldots$		
	$=\frac{1}{2}[2\ln\lvert\sec\theta\rvert+\tan\theta]$ or $\ln\lvert\sec\theta\rvert+\frac{1}{2}\tan\theta$ or $-\ln\cos\theta+\frac{1}{2}\tan\theta$ or $=\frac{1}{2}[-2\ln\lvert\cos\theta\rvert+\tan\theta]$	A1	1.1b
	$=\frac{1}{2}\left[2\ln\left\lvert\sec\left(\frac{\pi}{4}\right)\right\rvert+\tan\left(\frac{\pi}{4}\right)\right]-\frac{1}{2}[2\ln\lvert\sec(0)\rvert+\tan(0)]$ $=\left(\ln\left\lvert\sec\left(\frac{\pi}{4}\right)\right\rvert+\frac{1}{2}\tan\left(\frac{\pi}{4}\right)\right)-\left(\ln\lvert\sec 0\rvert+\frac{1}{2}\tan 0\right)$ $\left\{=\ln\sqrt{2}+\frac{1}{2}\right\}$	dM1	1.1b
	Area of triangle $=\frac{1}{2}xy=\frac{1}{2}\left(2\cos\frac{\pi}{4}\right)\left(2\sin\frac{\pi}{4}\right)=\ldots\left\{\frac{1}{2}\times\sqrt{2}\times\sqrt{2}=1\right\}$ The equation of the tangent is $r=\sqrt{2}\sec\theta$ then applies Area bounded of triangle $=\frac{1}{2}\int_0^{\frac{\pi}{4}}(\sqrt{2}\sec\theta)^2\ \{d\theta\}$	M1	1.1b
	Finds the required area = area of triangle – area bounded by the curve $=1-\left[\ln\sqrt{2}+\frac{1}{2}\right]$ May be seen within an integral $=\frac{1}{2}\int(\sqrt{2}\sec\theta)^2\ \{d\theta\}-\frac{1}{2}\int(1+\tan\theta)^2\ \{d\theta\}$	M1	3.1a
	$=\frac{1}{2}(1-\ln 2)$ * cso	A1*	2.1
		(6)	
	Alternative Area bounded by the curve $=\frac{1}{2}\int(1+\tan\theta)^2\ \{d\theta\}$ $=\frac{1}{2}\int(1+2\tan\theta+\tan^2\theta)\ \{d\theta\}$ let $u=\tan\theta\Rightarrow\frac{du}{d\theta}=\sec^2\theta$ Leading to $=\frac{1}{2}\int\frac{(1+2u+u^2)}{1+u^2}\{du\}=\frac{1}{2}\int 1+\frac{2u}{1+u^2}\{du\}=\ldots$	M1	3.1a
	$\frac{1}{2}[u+\ln(1+u^2)]$	A1	1.1b
	$\frac{1}{2}[(1+\ln(1+(1)^2))-(0+\ln 1)]$ or $\frac{1}{2}\left[\left(\tan\left(\frac{\pi}{4}\right)+\ln\left(1+\tan^2\left(\frac{\pi}{4}\right)\right)\right)-(\tan(0)+\ln(1+\tan^2(0)))\right]$ $\left\{=\frac{1}{2}\ln 2+\frac{1}{2}\right\}$	dM1	1.1b
	Area of triangle $=\frac{1}{2}xy=\frac{1}{2}\left(2\cos\frac{\pi}{4}\right)\left(2\sin\frac{\pi}{4}\right)=\ldots\left\{\frac{1}{2}\times\sqrt{2}\times\sqrt{2}=1\right\}$	M1	1.1b

	Finds the required area = area of triangle – area bounded by the curve $= 1 - \left[ln\sqrt{2} + \frac{1}{2}\right]$	M1	3.1a
	$= \frac{1}{2}(1 - ln\,2)$ *	A1*	2.1
		(6)	
(10 marks)			

Notes:

(a)
M1: Substitutes the equation of C into $x = r\cos\theta$ and differentiates to the required form
A1: Fully correct differentiation
dM1: Dependent on previous method mark. Sets their $\frac{dx}{d\theta} = 0$ and uses correct trig identities to find a value for θ. Alternatively substitutes $\theta = \frac{\pi}{4}$ into their $\frac{dx}{d\theta}$ and shows equals 0.
A1*: Shows that $r = 2$ and hence the polar coordinates $\left(2, \frac{\pi}{4}\right)$ from correct working

(b)
M1: Applies area $= \frac{1}{2}\int r^2\,\theta\ d\theta$, multiplies out, uses the identity $\pm 1 \pm tan^2\,\theta = sec^2\,\theta$ to get into an integrable form **and** integrates. Condone missing $d\theta$, limits are not required for this mark
A1: Correct integration. Note may include θ – θ if the one's were not cancelled earlier.
dM1: Dependent on the first method mark. Applies the limits of $\theta = 0$ and $\theta = \frac{\pi}{4}$ and subtracts the correct way round. Since substitution of the limit $\theta = 0$ is 0 so may be implied
M1: Correct method to find the area of triangle seen. This may be minimal but area = 1 only is M0, they need to show some method.
M1: Finds the required area = area of triangle – area bounded by the curve
A1*: Correct answer, with no errors or omissions. cso
Alternative
M1: Applies area $= \frac{1}{2}\int r^2\,\theta\ d\theta$, multiplies out, uses the substitution $u = tan\,\theta$ to get into an integrable form **and** integrates. Limits are not required for this mark
A1: Correct integration
dM1: Dependent on the first method mark. Applies the limits of $u = 0$ and $u = 1$ or substitutes back using $u = tan\,\theta$ and uses the limits $\theta = 0$ and $\theta = \frac{\pi}{4}$ and subtracts the correct way round. Since substitution of the limit $\theta = 0$ is 0 so may be implied
M1: Correct method to find the area of triangle
M1: Finds the required area = area of triangle – area bounded by the curve
A1*: Correct answer, with no errors or omissions. cso

Question	Scheme	Marks	AOs
8(a)	A complete method to use the scalar product of the direction vectors and the angle 120° to form an equation in a $$\frac{\begin{pmatrix}2\\a\\0\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}}{\sqrt{2^2+a^2}\sqrt{1^2+(-1)^2}}=\cos 1\,20$$	M1	3.1b
	$$\frac{a}{\sqrt{4+a^2}\sqrt{2}}=-\frac{1}{2}$$	A1	1.1b
	$2a=-\sqrt{4+a^2}\sqrt{2}\Rightarrow 4a^2=8+2a^2\Rightarrow a^2=4\Rightarrow a=...$	M1	1.1b
	$a=-2$	A1	2.2a
		(4)	
(b)	Any two of **i**: $-1+2\lambda=4$ (1) **j**: $5+\text{'their}-2\text{'}\lambda=-1+\mu$ (2) **k**: $2=3-\mu$ (3)	M1	3.4
	Solves the equations to find a value of $\lambda\left\{=\frac{5}{2}\right\}$ **and** $\mu\{=1\}$	M1	1.1b
	$r_1=\begin{pmatrix}-1\\5\\2\end{pmatrix}+\frac{5}{2}\begin{pmatrix}2\\\text{'their}-2\text{'}\\0\end{pmatrix}$ or $r_2=\begin{pmatrix}4\\-1\\3\end{pmatrix}+1\begin{pmatrix}0\\1\\-1\end{pmatrix}$	dM1	1.1b
	$(4,0,2)$ or $\begin{pmatrix}4\\0\\2\end{pmatrix}$	A1	1.1b
	Checks the third equation e.g. $\lambda=\frac{5}{2}$: LHS $=5-2\lambda=5-5=0$ $\mu=1$: RHS $=-1+\mu=-1+1=0$ therefore **common point/intersect/consistent/tick** or substitutes the values of λ and μ into the relevant lines and achieves the same coordinate	B1	2.1
		(5)	
(c)	Full attempt to find the minimum distance from the point of intersection (nest) to the plane (ground) E.g. Minimum distance $=\frac{\lvert 2\times\text{'4'}+(-3)\times\text{'0'}+1\times\text{'2'}-2\rvert}{\sqrt{2^2+(-3)^2+1)^2}}=...$ **Alternatively** $\mathbf{r}=\begin{pmatrix}\text{'4'}\\\text{'0'}\\\text{'2'}\end{pmatrix}+\lambda\begin{pmatrix}2\\-3\\1\end{pmatrix}$ $2(\text{'4'}+2\lambda)-3(\text{'0'}-3\lambda)+(\text{'2'}+\lambda)=2\Rightarrow$ $\lambda=...\left\{-\frac{4}{7}\right\}$	M1 A1ft	3.1b 3.4

	$\mathbf{r} = \begin{pmatrix} '4' \\ '0' \\ '2' \end{pmatrix} + '-\frac{4}{7}'\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{20}{7} \\ \frac{12}{7} \\ \frac{10}{7} \end{pmatrix}$ Minimum distance $= \sqrt{\left(2 \times -\frac{4}{7}'\right)^2 + \left(-3 \times -\frac{4}{7}'\right)^2 + \left(1 \times -\frac{4}{7}\right)^2} = \ldots$ $= \sqrt{\left('4' - '\frac{20}{7}'\right)^2 + \left('0' - '\frac{12}{7}'\right)^2 + \left('2' - '\frac{10}{7}'\right)^2} = \ldots$		
	$\frac{8}{\sqrt{14}}$ or $\frac{4\sqrt{14}}{7}$ or awrt 2.1	A1	2.2b
		(3)	
	Alternative Find perpendicular distance from plane to the origin $2x - 3y + z = 2$ $\|n\| = \sqrt{2^2 + (-3)^2 + 1^2} = \sqrt{14}$ shortest distance $= \frac{2}{\sqrt{14}}$ Find perpendicular distance from the plane containing the point of intersection to the origin $2x - 3y + z = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 10$ shortest distance $= \frac{10}{\sqrt{14}}$ Minimum distance $= \frac{10}{\sqrt{14}} - \frac{2}{\sqrt{14}}$	M1 A1ft	3.1b 3.4
	$\frac{8}{\sqrt{14}}$ or $\frac{4\sqrt{14}}{7}$ or awrt 2.1	A1	2.2b
		(3)	
(d)	For example Not reliable as the birds will not fly in a straight line Not reliable as angle between flights paths will not always be 120° Not reliable/reliable as the ground will not be flat/smooth Not reliable as bird's nest is not a point	B1	3.2b
		(1)	
			(13 marks)

Notes:

(a)
M1: See scheme, allow a sign slip and cos 60
A1: Correct simplified equation in a, cos 120 must be evaluated to – ½ and dot product calculated
Note: If the candidate states either $\left|\frac{a⃞b}{|a||b|}\right| = cos\,\theta$ or $\left|\frac{a}{\sqrt{4+a^2}\sqrt{2}}\right| = cos\,6\,0$then has the equation $\frac{a}{\sqrt{4+a^2}\sqrt{2}} = \frac{1}{2}$ award this mark. If the module of the dot product is not seen then award A0 for this equation.

dM1: Solve a quadratic equation for a, by squaring and solving an equation of the form $a^2 = K$ where $K > 0$ **A1:** Deduces the correct value of a from a correct equation. Must be seen in part (a) using the angle between the lines. **Alternative cross product method** **M1:** $\begin{vmatrix} 2 & a & 0 \\ 0 & 1 & -1 \end{vmatrix} = \sqrt{2^2 + a^2}\sqrt{1^2 + (-1)^2}\, sin\, 1\,20$ **A1:** $\sqrt{a^2 + 8} = \sqrt{4 + a^2}\sqrt{2}\frac{\sqrt{3}}{2}$ Then as above **Note If they use the point of intersection to find a value for *a* this scores no marks**
(b) **M1:** Uses the model to write down any two correct equations **M1:** Solve two equations simultaneously to find a value for μ and λ **dM1**: Dependent on previous method mark. Substitutes μ and λ into a relevant equation. If no method shown two correct ordinates implies this mark. **A1:** Correct coordinates. May be seen in part (c) **B1**: Shows that the values of μ and λ give the same third coordinate or point of intersection and draws the conclusion that the **lines intersect/common point/consistent** or tick. **Note:** If an incorrect value for a is found in part (a) but in part (b) they find that $a = -2$ this scores **B0** but all other marks are available
(c) This is M1M1A1 on ePen marking as M1 A1ft A1 **M1:** Full attempt to find the minimum distance from a point to a plane. Condone a sign slip with the value of d. **A1ft:** Following through on their point of intersection. Uses the model to find a correct expression for minimum distance from the nest to the ground **A1:** Correct distance **Alternative** **M1:** Find the shortest distance from a point to plane by finding the perpendicular distance from the given plane to the origin and the perpendicular distance from the plane contacting their point of intersection to the origin and subtracts **A1ft:** Following through on their point of intersection. Uses the model to find a correct expression for minimum distance from the nest to the ground **A1:** Correct distance
(d) **B1:** Comments on one of the models • Flight path of the birds modelled as a straight line • Angle between flight paths modelled as 120° • The bird's nest is modelled as a point • Ground modelled as a plane Then states unreliabl Any correct answer seen, ignore any other incorrect answers

Question	Scheme	Marks	AOs
9(a)(i)	$\frac{dy}{dx} = \ldots \cosh^{n-1} x \sinh x$ $\frac{d^2y}{dx^2} = \ldots \cosh^{n-2} x \sinh^2 x + \ldots \cosh^{n-1} x \cosh x$ **Alternatively** $y = \left(\frac{e^x+e^{-x}}{2}\right)^n$ leading to $\frac{dy}{dx} = \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-1}\left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^2y}{dx^2} = \ldots \left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + \ldots \left(\frac{e^x+e^{-x}}{2}\right)^n$	M1	1.1b
	$\frac{dy}{dx} = n\cosh^{n-1} x \sinh x$ $\frac{d^2y}{dx^2} = n(n-1)\cosh^{n-2} x \sinh^2 x + n\cosh^n x$ **Alternatively** $\frac{dy}{dx} = n\left(\frac{e^x+e^{-x}}{2}\right)^{n-1}\left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^2y}{dx^2} = n(n-1)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + n\left(\frac{e^x+e^{-x}}{2}\right)^n$	A1	2.1
	$\frac{d^2y}{dx^2} = n(n-1)\cosh^{n-2} x\,(\cosh^2 x - 1) + n\cosh^n x$	M1	2.1
	$\frac{d^2y}{dx^2} = n^2\cosh^n x - n(n-1)\cosh^{n-2} x$* cso	A1*	1.1b
		(4)	
(a)(ii)	$\frac{d^3y}{dx^3} = \ldots \cosh^{n-1} x \sinh x - \ldots \cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4}$ $= \ldots \cosh^{n-2} x \sinh^2 x + \ldots \cosh^n x - \ldots \cosh^{n-4} x \sinh^2 x - \ldots \cos$	M1	1.1b
	$\frac{d^3y}{dx^3} = n^3\cosh^{n-1} x \sinh x - n(n-1)(n-2)\cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4} = n^3(n-1)\cosh^{n-2} x \sinh^2 x + n^3\cosh^n x$ $-n(n-1)(n-2)(n-3)\cosh^{n-4} x \sinh^2 x - n(n-1)(n$ $-2)\cosh^{n-2} x$	A1	1.1b
		(2)	
	Alternative 1 using $\frac{d^2y}{dx^2} = n^2y - n(n-1)\cosh^{n-2} x$ leading to $\frac{d^3y}{dx^3} = n^2\frac{dy}{dx} - \ldots \cosh^{n-3} x \sinh x$ $\frac{d^4y}{dx^4} = n^2\frac{d^2y}{dx^2} - \ldots \cosh^{n-4} x \sinh^2 x - \ldots \cosh^{n-2} x$	M1	1.1b

	$\frac{d^3y}{dx^3} = n^2\frac{dy}{dx} - n(n-1)(n-2)\cosh^{n-3}x\sinh x$ $\frac{d^4y}{dx^4} = n^2\frac{d^2y}{dx^2} - n(n-1)(n-2)(n-3)\cosh^{n-4}x\sinh^2 x$ $- n(n-1)(n-2)\cosh^{n-2}x$	A1	1.1b
		(2)	
	Alternative 2 $y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2\cosh^n x - n(n-1)\cosh^{n-2}x$ $y = \cosh^{n-2}x \Rightarrow \frac{d^2y}{dx^2} = \ldots\cosh^{n-2}x - \ldots\cosh^{n-4}x$ $\frac{d^4y}{dx^4} = n^2\left[n^2\cosh^n x - n(n-1)\cosh^{n-2}x\right]$ $- n(n-1)\left[\ldots\cosh^{n-2}x - \ldots\cosh^{n-4}x\right]$	M1	1.1b
	$y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2\cosh^n x - n(n-1)\cosh^{n-2}x$ $y = \cosh^{n-2}x \Rightarrow \frac{d^2y}{dx^2}$ $= (n-2)^2\cosh^{n-2}x - (n-2)(n-3)\cosh^{n-4}x$ $\frac{d^4y}{dx^4} = n^2\left[n^2\cosh^n x - n(n-1)\cosh^{n-2}x\right]$ $- n(n-1)\left[(n-2)^2\cosh^{n-2}x - (n-2)(n-3)\cosh^{n-4}x\right]$	A1	1.1b
		(2)	
	Alternative 3 Using $\frac{d^2y}{dx^2} = n^2\left(\frac{e^x+e^{-x}}{2}\right)^n - n(n-1)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ leading to $\frac{d^3y}{dx^3} = \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-1}\left(\frac{e^x-e^{-x}}{2}\right) - \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-3}\left(\frac{e^x-e^{-x}}{2}\right)$ $\frac{d^4y}{dx^4} = \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ $- \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-4}\left(\frac{e^x-e^{-x}}{2}\right)^2 - \ldots\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$	M1	1.1b
	$\frac{d^3y}{dx^3} = n^3\left(\frac{e^x+e^{-x}}{2}\right)^{n-1}\left(\frac{e^x-e^{-x}}{2}\right) - n(n-1)(n-2)\left(\frac{e^x+e^{-x}}{2}\right)^{n-3}\left(\frac{e^x-e^{-x}}{2}\right)$	A1	1.1b

	$\frac{d^4y}{dx^4} = n^3(n-1)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}\left(\frac{e^x-e^{-x}}{2}\right)^2 + n^3\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$ $-n(n-1)(n-2)(n-3)\left(\frac{e^x+e^{-x}}{2}\right)^{n-4}\left(\frac{e^x-e^{-x}}{2}\right)^2 - n(n-1)(n-2)\left(\frac{e^x+e^{-x}}{2}\right)^{n-2}$		
		(2)	
(b)	When $x = 0$ $y = 1,\quad y' = 0,\quad y'' = n^2 - n(n-1),\quad y^{(3)} = 0,$ $y^{(4)} = n^3 - n(n-1)(n-2)$ Uses their values in the expansion $y = y(0) + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y^{(3)}(0) + \frac{x^4}{4!}y^{(4)}(0) + \ldots$	M1	1.1b
	$y = 1 + \frac{nx^2}{2} + \frac{(3n^2-2n)x^4}{24} + \ldots$ cso	A1	2.5
		(2)	
			(8 marks)

Notes:

(a)(i)

M1: Uses the chain rule and product rule to find the first and second derivatives which must be of the required form, condone sign slips

Alternatively uses the exponential definition and uses the chain rule and product rule to find the first and second derivatives which must be of the required form.

A1: Correct unsimplified first and second derivatives, may be in exponential form.

M1: Uses the identity $\pm cosh^2 x \pm sinh^2 x = 1$

A1*: Achieves the printed answer with no errors or omissions e.g. missing x's

(a)(ii)

M1: Uses the chain rule and product rule to find the third and fourth derivatives which must be of the required form, condone sign slips

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 1

M1: Using $\frac{d^2y}{dx^2} = n^2y - n(n-1)\cosh^{n-2} x$ to find the third and fourth derivatives which must be of the required form, condone sign slips

A1: Correct fourth derivative, does not need to be simplified ISW

Alternative 2

M1: Using $y = \cosh^n x \Rightarrow \frac{d^2y}{dx^2} = n^2 \cosh^n x - n(n-1)\cosh^{n-2} x$

$y = \cosh^{n-2} x \Rightarrow \frac{d^2y}{dx^2} = \ldots\cosh^{n-2} x - \ldots\cosh^{n-4} x$ leading to

$$\frac{d^4y}{dx^4} = n^2\left[n^2 \cosh^n x - n(n-1)\cosh^{n-2} x\right] - n(n-1)\left[\text{their} \frac{d(\cosh^{n-2} x)}{dx}\right]$$ **A1:** Correct fourth derivative, does not need to be simplified ISW **Alternative 3** **M1:** Uses the exponential definition and uses the chain rule and product rule to find the third and fourth derivatives which must be of the required form. **A1:** Correct fourth derivative, does not need to be simplified ISW
(b) **M1:** Attempts the evaluation of all four of their derivatives at $x = 0$ and applies the Maclaurin formula with their values. Note that $y^{(1)}(0) = 0$ and $y^{(3)}(0) = 0$ may be implied as they will have a multiple of sinh0. If their $y^{(3)}(0) \neq 0$ they allow this mark for their first 3 non-zero terms **A1:** Correct simplified expansion from correct derivatives cso

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